Now, for inviscid low-Mach-number flow

$$p=\rho+0(M_R^2) \tag{9a}$$

Therefore, to $0(M_R^2)$

$$u\cdot\nabla\rho=u\cdot\nabla p \tag{9b}$$

$$\rho_t=p_t \tag{9c}$$

and

$$u\cdot\nabla p=-(1/M_R)^2\nabla\cdot u \tag{9d}$$

From Eq. (9d) we conclude that $\nabla\cdot u=0(M_R^2)$. If we combine Eqs. (9b), (9c), and (6a), we get Eq. 7. This means, that to $0(M_R^2)$, the energy and continuity equations are identical in inviscid regions of the flow.

The above analysis partially explains why conventional compressible flow computations break down at low Mach numbers—in the steady state, the continuity equation and the inviscid part of the energy equations become very similar, leading to ill conditioning of the numerics (also, the use of p and ρ instead of p' and ρ' and the use of total rather than perturbation scaling quantities in conventional compressible computations produce a high noise-to-signal ratio).

Let us next consider the viscous case. Only if there is no heat addition, the walls are cold and k is small, can the terms in the energy equation involving the temperature be safely neglected. Doing so would maintain the uncoupling of the continuity from the other two equations.

V. Conclusions

The method of pseudocompressibility can be regarded as one of computing incompressible flows by replacing the incompressible continuity equation with part of the low-Mach-number energy equation. For the time-accurate calculation of unsteady flows, the terms absent from the pseudocompressible continuity/energy equation are, in general, not negligible; numerical experiments should be carried out to compare the differences between using Eqs. (1) and (5) or Eq. (6).

To use Eqs. (5b) and (5c), or (6b) and (6c), one should choose a value of M_R low enough so that the flow is in effect incompressible (truly incompressible flows do not exist in nature anyway; it is assumed that when the Mach-number-like parameter of any Navier-Stokes flow of a perfect gas, a liquid, or whatever, is sufficiently low, then this flow is effectively similar to a truly incompressible one), but not so low that the term $1/M_R^2$ in Eqs. 5 is troublesome or that the approximate-factorization error[2,3] is too large. If the numerical algorithm used to solve Eqs. (5) does not make use of approximate factorization or, if it corrects for it, then the upper bound for $1/M_R^2$ discussed in Refs. 2 and 3 can be relaxed and lower Mach numbers can be safely utilized.

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Turbulent Mixing in Nonsteady Jets

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Nomenclature

d	= nozzle diameter
K_1,K_2	= constants
$L(t)$	= instantaneous flame length
Re	= Reynolds number
t	= time
U_{cl}	= jet centerline speed
Ve	= jet entrainment speed
$U_j(t)$	= nozzle exit speed
x	= downstream distance from nozzle exit
$\Gamma(t)$	= circulation
$\delta(t)$	= instantaneous maximum visible flame width
ϕ	= equivalence ratio
Ω	= vorticity

Introduction

THE mixing behavior of the steady jet has been extensively studied. The mean concentration of any conserved scalar ejected from the jet nozzle declines as x^{-1} as the jet mixes with the ambient fluid, where x is the downstream distance. Therefore, one expects that any molecular scale mixing occurring as the fluid proceeds downstream will initially produce mixtures rich in injected fluid near the nozzle and progressively leaner mixtures further downstream. Such behavior has been observed in laboratory experiments.[1-3] It is clear from this that the steady jet mixes the two fluids together over a wide range of mixture ratios. If some mixture ratios are undesirable, the steady jet is assured of producing at least some undesirable mixing. Thus, if the mixing rate could be modified by some means to preferentially mix fluid at a single mixture ratio, rather than the broad range covered by the steady jet, then the amount of mixing at the undesirable mixture ratios could be reduced.

We propose that a nonsteady jet may modify the molecular mixing rate by altering the ratio of ambient to nozzle fluid ingested into turbulent vortices. The notion is fairly simple. Consider the starting jet illustrated in Fig. 1. If the nozzle exit velocity $U_j(t)$ is a step function in time, a toroidal starting vortex is produced. As it convects away from the nozzle, it grows, partially by entrainment of ambient fluid and partially by incorporation of overtaking nozzle fluid from the rear. While, to our knowledge, the ratio of the two contributing components has not been measured for the jet, the related quantity for the starting buoyant plume is reported by Turner[4] to be approximately unity. That is, about equal rates of ambient and nozzle fluid are incorporated into the starting vortex for the plume.

In the jet, the incorporation ratio of ambient to nozzle fluid may be changed by nonsteady effects. At any instant, the vortex has circulation $\Gamma(t)$ and characteristic dimension $\delta(t)$. Thus, it has a finite-volume entrainment appetite of order $\Gamma\delta(t)$. If the flow out of the nozzle is continually accelerated so that the vortex engulfment[5] appetite is completely satisfied by this stream, then negligible ambient fluid would be en-

Received May 23, 1985; presented as Paper 86-0042 at the AIAA 24th Aerospace Sciences Meeting, Reno, NV, Jan. 6-9, 1986; revision received March 5, 1986.

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trained. The vortex would grow solely by incorporation of nozzle fluid from behind (A in Fig. 1). With no ambient fluid (B in Fig. 1) entering it, little mixing could occur within the vortex. Since previous work[6,7] has shown that little mixing occurs at the edges of the structure, one might expect dramatic reductions in the mixing rate. This argument can be extended to the other vortices in the jet, in addition to the starting vortex.

From dimensional considerations, the local entrainment velocity Ve must be proportional to the local centerline speed $U_{c\ell}$ of the steady, unforced jet, there being no other appropriate dimensional quantity. So

$$V_e = K_1 U_{c\ell} \tag{1}$$

where K_1 is a dimensionless constant. Morton et al.[8] used this argument to analyze plumes and thermals.

But if an externally imposed time scale is present, then the entrainment velocity to first order may depend on the time derivative as well,

$$V_e = K_1 U_{c\ell} + \frac{K_2}{\Omega}\frac{\partial}{\partial t} U_{c\ell} \tag{2}$$

where K_2 is a dimensionless constant and $1/\Omega$ is some time. The most natural choice for the latter is the rotation period of the large-scale vortices. This line of reasoning suggests that Ve is modified from its steady value only so long as, at each rotation, the increment in global vorticity due to the acceleration is significant compared to the existing vorticity Ω. To maintain this condition forever, the acceleration must be exponential. If the acceleration is less than exponential (for example, merely linear), the nonsteady effects would be expected to disappear after a time when the first term on the right-hand side of Eq. (2) dominates the second. The flow would relax back to its quasisteady behavior, even though the jet is still accelerating.

It should be noted that other possibilities have been recognized. Morton[9] and Telford[10,11] proposed that for a flow not in equilibrium, the entrainment velocity would depend on the Reynolds stress or turbulence intensity rather than the mean velocity.

Steady Jet

Weddell[12] observed liquid jets with pH indicators to determine their mixing characteristics. He chose an indicator, phenolphthalein, with appropriate acid and base concentrations such that the indicator disappeared after it left the nozzle and mixed with a certain, prescribed amount of ambient fluid. The volume ratio of ambient to injected fluid necessary to affect the indicator transition is termed equivalence ratio ϕ. The chemistry is fast compared with the fluid mechanical mixing.

Weddell's results can be summarized as follows: the phenolphthalein disappears at a distance, called "flame length" L, which depends only on the nozzle diameter d and equivalence ratio ϕ at high jet Reynolds numbers, $Re \geq 3 \times 10^3$. He found that

$$(L - s)/d = 10\phi$$

where s is the "break point" or virtual origin near the nozzle where the jet begins to spread at the turbulent growth angle. Note that the steady flame length is completely independent of

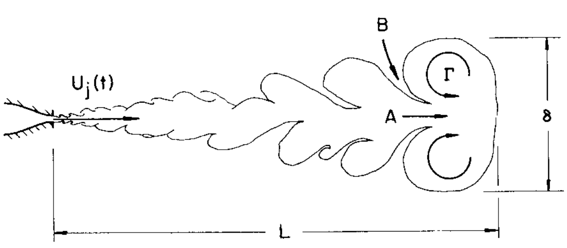

Fig. 1 Flow geometry.

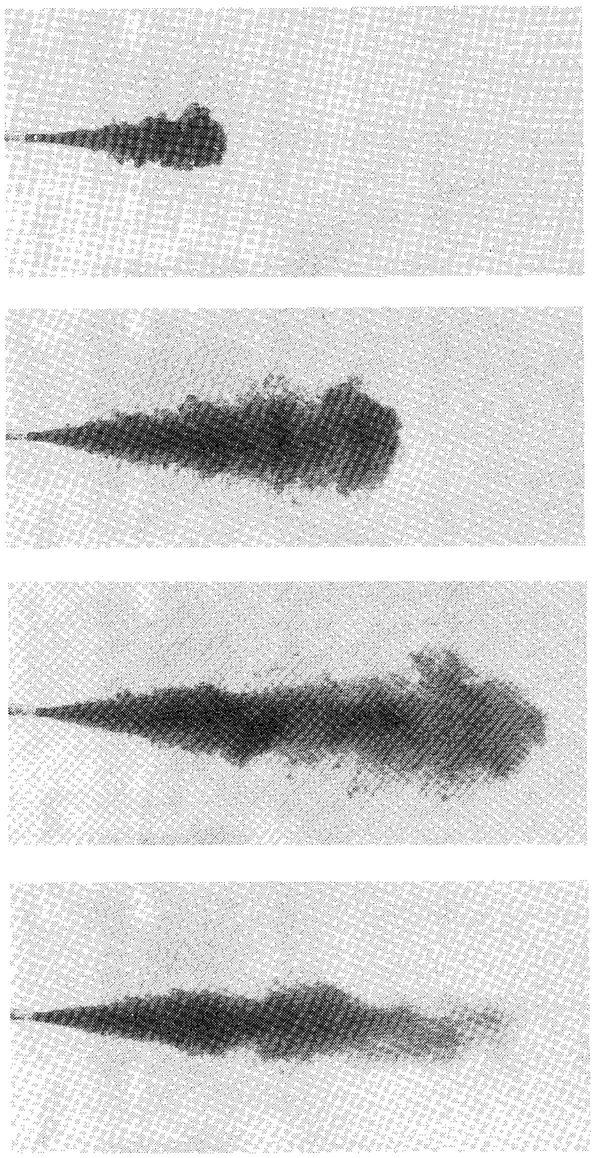

Fig. 2 Accelerating jet flame.

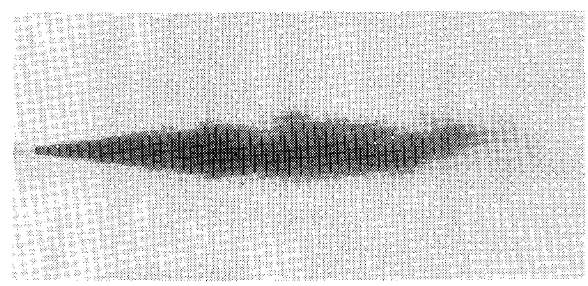

Fig. 3 Steady jet flame.

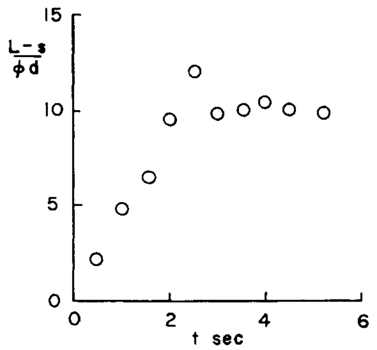

Fig. 4 Time history of the visible flame length.

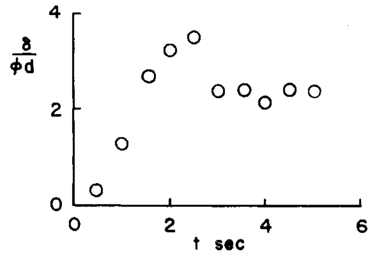

Fig. 5 Time history of the visible flame width.

jet speed, consistent with the proportional relationship in Eq. (1).

The liquid jet results of Weddell have been compared to gas flame results.[1] Rather surprisingly, the flame length for liquid jets and gas flames are identical at high Reynolds number. Experimental results in liquid flows evidently have general validity in gas flows as well. Physical reasons for this are discussed by Broadwell.[13]

Experiments

The experiments were conducted in a large lucite water tank of 2 m^3 volume. The jet fluid was injected into the ambient fluid near the top of the tank though a nozzle of 0.3 cm exit diameter d, after an area contraction ratio of four in the nozzle. The flame length was measured by Weddell's technique. Using a modulated valve, the nozzle jet speed increased from zero as an approximately linear function of time, with typical Reynolds numbers of about 10^4 during the middle of the acceleration. Buoyancy effects were negligible. A second valve with an approximately quadratic nozzle speed function of time gave essentially identical results.

Results and Discussion

A sequence of photographs of the accelerating jet is shown in Fig. 2 for the case of an equivalence ratio of seven. A steady jet is shown in Fig. 3 for otherwise identical conditions. A comparison of Figs. 2 and 3 (and many others like them) demonstrates that the nonsteady flame is momentarily somewhat longer and considerably wider than the mean dimensions of the corresponding steady flame. From many such sequential photographs in a typical run, an x-t diagram of the visible leading edge of the jet is shown in Fig. 4. The visible jet grows in length with time until it reaches a maximum length. Then it contracts slightly to the steady-state value, even though it is still accelerating. The maximum observed flame length is about 25% greater than its steady-state value.

More dramatic is the behavior of the maximum visible width $\delta(t)$. The time evolution of δ is plotted in Fig. 5. Here the maximum flame width exceeds the steady flame width by about 50%. Evidently, the nonsteady effects have considerably modified the mixing process.

It is possible that the observations could be explained by some other effect. For example, the acceleration might permanently transform the vortex structure, so that the mixing rate is modified even long after the acceleration is over. At issue is the relaxation time constant, or number of vortex revolutions for it to return to the undisturbed steady state. Another possibility is that the starting vortex always mixes at a slower rate than the subsequent vortices by virtue of its unique position. Both possibilities can be tested by observing the flame when the nozzle flow is a step function in time, rather than a linear or quadratic ramp. The initial tests of such a flame indicate only a modest increase in dimension over the steady flame, much less than in the linear or quadratic case reported here. More detailed results will be presented in a full paper.

It is interesting to speculate on the nonsteady modification of the concentration field to that of the steady jet, the latter carefully measured by Dahm and Dimotakis.[7] Since by definition the starting vortex is the first one, it cannot be subject to the trailing wings of a predecessor vortex as they infer from their observations of the steady jet. Instead, the starting vortex seems to exhibit a relatively uniform internal mixture all the way out to its edge, since the indicator in it does not disappear until the entire structure has entrained sufficient volume of ambient fluid, at which point the whole structure disappears. This may explain why the maximum flame length L is only about 25% greater than the steady flame length, while the maximum flame width is 50% greater.

Conclusions

In a simple experiment, the mixing characteristics of an accelerating turbulent jet in aqueous solution has been compared to those of the more closely examined steady jet. Using pH indicators, the maximum visible extent of accelerating "flames" was found to be about 25% greater than steady flames and about 50% wider. The mixing process in the jet is significantly modified by nonsteady effects. The entrainment velocity depends not only on the jet velocity, but also on its time derivative. Experiments on exponential jets, where nonsteady effects are anticipated to be more persistent, will be reported in the future. Those experiments will address the question of nonsteady effects on not only the starting vortex, but subsequent ones as well.[14]

Acknowledgments

The authors wish to thank Hamid Johari for many useful discussions and Otto Brask for his help in constructing the apparatus. The original ideas evolved under support of Contract NSF ATM-8314556.

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Effect of Reynolds Number on the Structure of Recirculating Flow

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Introduction

IN this work, we apply the random vortex method[1] to compute the unsteady flow over a rearward-facing step at a wide range of Reynolds number. The simulation is restricted to two-dimensions to limit the computational efforts, and, hence, high Reynolds number effects are manifested only by the growth of perturbations into cyclic variations and the formation of small scales. Attention is focused on predicting the effect of Reynolds number on the dynamics and structure of the flowfield in terms of processes of vortex eddy formation and interaction.

The use of vortex elements to represent the vorticity field and follow its dynamics in a numerical simulation is a natural way to overcome the convective nonlinearity of the Navier-Stokes equations without introducing excessive numerical diffusion. Random walk, used to simulate diffusion in the scheme, is complementary to this representation since it introduces the effect of molecular viscosity without impeding any of the attributes of inviscid vortex methods.[2] The scheme is grid-free and self-adaptive, and computational elements conglomerate around large-velocity gradients to provide high resolution without an unbounded increase in computational effort. Since the Reynolds number controls the relative size of diffusive transport with respect to convective transport, it plays the same role in the numerical algorithm as it does in physics: it controls the balance between convection and diffusion. Random walk introduces a wide spectrum of noise that can excite natural flow instabilities during transition without inhibiting their growth. For more discussion on the algorithm, see Ghoniem and Gagnon[3] and Sethian and Ghoniem.[4] The computer code employed is a modified version of MIMOC.[5]

Solution

Computations were performed for a large matrix of numerical parameters for each Reynolds number to check the convergence of the results.[4] Here, we report on results for a single set of parameters, namely, time step $\Delta t = 0.1$, length of vortex sheets $h = 0.4$, circulation of elementary vortices $\Gamma = 0.025$, and computational domain length $X = 10$. Variables are normalized with respect to the appropriate combination of incoming flow velocity U and step heights S. Four Reynolds numbers, defined as $R = US/\nu$, where ν is the kinematic viscosity, are selected: $R = 50$, 125, 500, 5000. For all values of R, the flow reached a stationary state during the first 500 time steps. The data are analyzed for time steps 501–1000.

Vorticity Structure

Instantaneous streamlines are obtained by tracing a set of particles injected at the inlet of the channel and at areas of slow flow until they leave the channel or form closed trajectories. The velocity field is averaged over 10 time steps to remove small noise.

$R = 50$

Figure 1 shows that the recirculation zone is formed of one large eddy, which remains stationary at the corner of the step. The eddy shape can be approximated by an ellipse with the major axis the size of 3S. The recirculation zone maintains a constant size and stays as one structure at all times, indicating limited interactions with the freestream. Thus, convective mixing between the freestream and the recirculating bubble is limited, and all of the exchange takes place by diffusion across the dividing streamline. Stability is clearly observed as small oscillations of the eddy, introduced by the stochastic simulation of diffusion, are damped. The flow can be characterized as viscous and steady.

The computed length of the recirculating zone is 3S, in agreement with the values reported in Denham and Patrick[6] and Armaly et al.[7] Flow visualization studies for a starting flow behind a step without an opposite wall, reported by Honji,[8] show the formation of one eddy at low Reynolds number and confirm its stability.

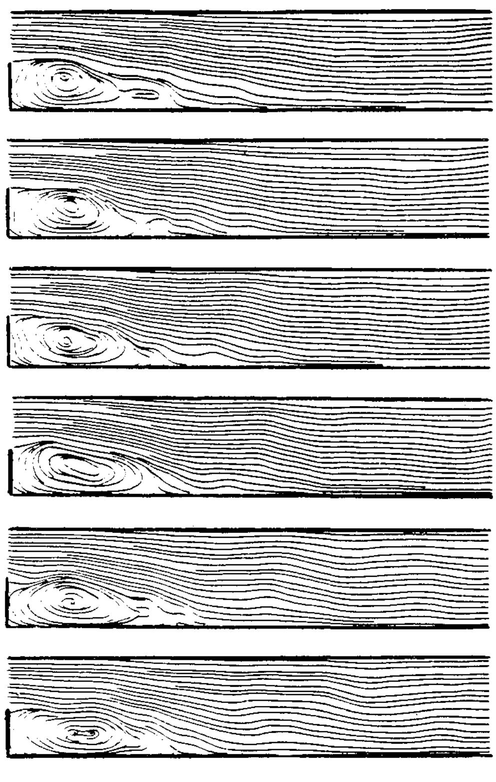

Fig. 1 Instantaneous streamline plots plotted every 10 time steps, starting at $t = 84.8$, $R = 50$.

Presented as Paper 85-0146 at the AIAA 23rd Aerospace Sciences Meeting, Reno, NV, Jan. 14-17, 1985; received Aug. 23, 1985; revision received June 16, 1986.

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